INTEGRALES POR SUSTITUCION

1

$$\int \sqrt{3\,x+5}\;dx \;= I$$

Sea: $du = 3\,x + 5$

$$du = 3\,dx \;=\; dx = \frac{du}{3}$$

Al sustituir tenemos:

$$I = \int u^{\frac{1}{2}} \frac{du}{3} \;= \frac{1}{3} \int u^{\frac{1}{2}}\, du$$

$$I = \frac{1}{3}\left(\frac{u^{\frac{3}{2}}}{\frac{3}{2}}\right) + \;C \;=\; \frac{2}{9} u^{3/2} + \;C$$

$$I = \;\frac{2}{9}(3x+2)^{3/2} + \;C$$

2

$$\int \frac{xdx}{(x^2 +\; 4)^2} \;= I$$

Sea: $u = x^2 + 4$

$$du = 2x\;dx \;=\; xdx = \frac{du}{2}$$

Al sustituir tenemos:

$$I = \int \frac{du}{2u^2} \;= \frac{1}{2} \int u^{-2}\,du$$

$$I = \frac{1}{2}\left(\frac{u^{-1}}{-1}\right) + \;C \;=\; \frac{1}{2} u^{-1} + \;C$$

$$I = \;\frac{1}{2}(x^2+4)^{-1} + \;C$$

$$I = \;\frac{1}{2(x^2+4)} + \;C$$

3

$$\int \frac{sen\ dx}{\sqrt{1+2\cos x}} = I$$

Sea: $u = 1 + 2\cos x$

$$du = -2\ sen\ x dx \ = \ sen\ x\ dx = -\frac{du}{2}$$

Al sustituir tenemos:

$$I = \int \frac{-du}{2u^{1/2}} = -\frac{1}{2}\int \frac{du}{u^{1/2}}$$

$$I = -\frac{1}{2}\int u^{-1/2}du = -\frac{1}{2}\left(\frac{u^{\frac{1}{2}}}{\frac{1}{2}}\right) + C$$

$$I = -u^{1/2} + C = -(1+2\cos x)^{1/2} + C$$

$$I = -\sqrt{1+2\cos x} + C$$

4

$$\int \frac{xdx}{\sqrt{x+1}} = I$$

Sea: $u = \sqrt{x+1}$

$$u^2 = x + 1 \ = \ x = u^2 - 1$$

$$2udu = dx$$

Al sustituir tenemos:

$$I = \int \frac{(u^2-1)2udu}{u} = 2\int (u^2-1)du$$

$$I = 2\left(\frac{u^2}{3} - u\right) + C = 2u\left(\frac{u^2 - 3}{3}\right) + C$$

$$I = \frac{2}{3}u(u^2 - 3) + C = \frac{2}{3}\sqrt{x+1}(x+1-3) + C$$

$$I = \frac{2}{3}\sqrt{x+1}(x-2) + C$$

5

$$\int x^2\sqrt{2-x}\ \ dx\ = I$$

Sea: $u=\sqrt{2-x}$

$$u^2 = 2-x = \quad x = 2-u^2$$
$$dx = -2udu$$

Reemplazando:

$$I = \int (2-u^2)^2(-2udu)(u)\quad = -2\int (2-u^2)udu\,(u^2)$$

$$I = -2\int (4-4u^2+\ u^4)u^2du$$

$$I = -2u^3\left(\frac{4}{3}-\frac{4}{5}u^2+\ \frac{u^4}{7}\right)+\ C$$

$$I = -2u^3\left(\frac{140-84u^2+15u^4}{105}\right)+\ C$$

$$I = -\frac{2}{105}u^3(140-84u^2+15u^4)+\ C$$

$$-\frac{2}{105}\left(\sqrt{2-x}\ \right)^3\left(140-84\left(\sqrt{2-x}\right)^2+15\left(\sqrt{2-x}\right)^4\right)+\ C$$

$$-\frac{2}{105}(2-x\ )^{3/2}(140-84(2-x)+15(2-x)^2)+\ C$$

6

$$\int\frac{xdx}{\sqrt{x^2+9}}\ = I$$

Sea: $u = x^2$+9

$$du = 2xdx\ = \quad xdx = \frac{du}{2}$$

Sustituyendo tenemos:

$$I = \int\frac{du}{2u^{1/2}}\ = 2\int u^{-1/2}\,du$$

$$I = \frac{1}{2}\left(\frac{u^{1/2}}{\left(\frac{1}{2}\right)}\right)+C$$

$$I = u^{1/2}+\ C$$

$$I = \sqrt{x^2+9}+\ C$$

(7)

$$\int \left(\frac{\sqrt{x}}{\sqrt{x}+1}\right) dx = I$$

Sea: $u = \sqrt{x} + 1 = \boxed{x = u - 1}$

$$(u-1)^2 = (\sqrt{x})^2$$

$$x = (u-1)^2$$

$$\boxed{dx = 2(u-1)du}$$

$$I = \int \left(\frac{u-1}{u}\right)^2 (2)(u-1)du$$

$$I = \int \frac{2(u-1)^2}{u}\, du$$

$$I = 2\int \frac{u^2 - \ 2u + 1}{u}\, du$$

(8)

$$\int x^2\sqrt{1+x}\ \ dx \ \ = I$$

Sea: $u = \sqrt{1+x}$

$$u^2 = 1 + x = \quad x = u^2 - 1$$
$$dx = 2udu$$

Reemplazando:

$$I = \int (u^2-1)^2 u(2u)du \quad = 2 \int (u^4 - 2u^2 + 1)u^2 du$$

$$I = 2 \int (u^6 - 2u^4 + u^2)du$$

$$I = 2\left(\frac{u^7}{7} + \frac{2u^5}{5} + \frac{u^3}{3}\right) + \ C$$

$$I = 2u^3\left(\frac{15u^4 - 42u^2 + 35}{105}\right) + \ C$$

$$-\frac{2}{105}(1+x)^{3/2}[15(1+x)^2 - 42(1+x) + 35] \ + \ C$$

9

$$\int (x+2)^2\sqrt{1+x}\ \ dx\ \ = I$$

Sea: $u = \sqrt{1+x}$

$$u^2 = 1 + x = \ \ dx = 2udu$$

Reemplazando:

$$I\ = \int\ (u^2-1)^2 u(2u)\ du$$

$$I = 2\int (u^4+2u^2+1)u^2 du$$

$$I = 2\int (u^6+2u^4+u^2)du$$

$$I = 2\left(\frac{u^7}{7}+\frac{2u^5}{5}+\frac{u^3}{3}\right)+\ C$$

$$I = 2u^3\left(\frac{15u^4+42u^2+35}{105}\right)+\ C$$

$$-\frac{2}{105}(1+x)^{3/2}[15(1+x)^2-42(1+x)+35]\ +\ C$$

10

$$\int \sqrt{1+\sqrt{x}}\ \ dx\ \ = I$$

Sea: $u = \sqrt{1+\sqrt{x}}$

$$u^2 = 1+\sqrt{x} = \qquad u^2 - 1 = \sqrt{x}$$

$$(u^2-1)^2 = x$$

$$2(u^2-1)(2u)du = dx$$

Reemplazando:

$$I = \int u[2(u^2-1)^2(2u)]\ du$$

$$I = 4\int u^2(u^2-1)du$$

$$I = 4\int (u^4-u^2)du$$

$$I = \left(\frac{u^4}{5} - + \frac{u^3}{3}\right)+\ C$$

$$I = 4u^3\left(\frac{3u^2-5}{15}\right)+\ C$$

$$4\left(1+\sqrt{3}\right)^{3/2}\left[3(1+\sqrt{x})-5\right]\ +\ C$$

11

$$\int e^x\sqrt{1+e^x}\ \ dx\ \ = I$$

Sea: $u = 1 + e^x$

$$du = e^x\ dx$$

Reemplazando tenemos:

$$I\ =\ \int\ u^{1/2}\ du$$

$$I = 2\ \int (u^4 + 2u^2 + 1)u^2 du$$

$$I = \frac{u^{3/2}}{\frac{3}{2}} +\ C$$

$$I = \frac{2}{3}u^{3/2} +\ C$$

$$I\ = \frac{2}{3}(1 + e^x)^{3/2}\ +\ C$$

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$$\int tan^3x\ sec^2x\ \ dx\ \ = I$$

Sea: $u = \tan\ x$

$$du = sec^2xdx$$

Reemplazando tenemos:

$$I =\ \ \int u^3du = \qquad I = \frac{u^x}{4} +\ C$$

$$I = \frac{tan^4x}{4}\ + C$$

13

$$\int tan^3x\ sec^2x\ \ dx\ \ = I$$

Sea: $u = \tan\ x$

$$du = sec^2xdx$$

Reemplazando tenemos:

$$I =\ \ \int u^3du = \qquad I = \frac{u^x}{4} +\ C$$

$$I = \frac{tan^4x}{4}\ + C$$

14 Calcular

$$\int_1^5 \frac{x}{\sqrt{2x-1}}\ dx\ = I$$

Sea: $u = \sqrt{2x-1}$

$u^2 = 2x - 1 \ = \ x = \frac{u^2+1}{2}$

$udu = dx$

Ahora, hagamos el cambio de límites de integración:

- ✓ Límite superior

Si x=5: $u = \sqrt{10-1} = 3$

- ✓ Límite inferior

Si x=1: $u = \sqrt{2-1} = 1$

Haciendo la sustitución, obtenemos

$$I = 2\int_1^3 \frac{1}{2}\left(\frac{u^2 + 1}{u}\right) udu$$

$$I = \frac{1}{2}\int_1^3 (u^2 + 1)\, du = \frac{1}{2}\left(\frac{u^3}{3} + u\right)_1^3$$

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$$\int \frac{x}{x^2+1}\ dx\ = I$$

Sea: $u = x^2 + 1$

$$du = 2xdx \quad = \quad xdx = \frac{du}{2}$$

Reemplazando tenemos:

$$I = \int \frac{du}{2u} = \qquad I = \frac{1}{2}\int \frac{du}{u} = \frac{1}{2}\lim(u) + C$$

$$I = \frac{1}{2}\lim(x^2+1) + C$$

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$$\int \frac{x^2+x+1}{x^2+1}\ dx\ = I$$

Al hacer la división se obtiene

$$\frac{x^2+x+1}{x^2} = 1 + \frac{x}{x^2+1}$$

Por lo tanto

$$I = \int \left(1 + \frac{x}{x^2+1}\right) dx = \quad \int dx + \int \frac{x}{x^2+1} dx$$

$$I = \int x + \frac{1}{2}\lim(x^2+1) + C$$

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$$\int \frac{2x}{(x+\ 1)^2}\ dx\ \ = I$$

Sea: $u = x + 1$

$x = u - 1$ $du = dx$

Al hacer la sustitución tenemos:

$$I = \int \frac{2(u-1)}{u^2} du$$

$$I = \int \left(\frac{2}{u} - \frac{2}{u^2}\right) du$$

$$I = \int \frac{du}{u} - 2\int u^{-2} du$$

$$I = 2\lim(u) - \frac{2u^{-1}}{(-1)} + \ C$$

$$I = 2\lim(u) - \frac{2}{u} + \ C$$

$$I = 2\lim(x+1) + \frac{2}{x+1} + \ C$$

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$$\int \frac{x^2 + 2x}{(x+\ 1)^2}\ dx\ \ = I$$

Al hacer el binomio y dividir se tiene

$$(x+1)^2\ = 1 - \frac{1}{x^2+2x+1}$$

Al sustituir

$$I = \ \int \left(1 - \frac{x}{(x+1)^2}\right) dx = \ \int dx - \int \frac{dx}{(x+1)^2}$$

$$I = \ x - \int \frac{dx}{(x+1)^2}$$

Sea: $I_2 = \int \frac{dx}{(x+1)^2}$

$u = x + 1$
$du = dx$

$$I_2 = \int \frac{du}{u^2} = \int u^{-2} du$$

$$I_2 = \frac{u^{-1}}{-1} + \ C$$

$$I_2 = -\frac{1}{u} + \ C \quad = -\frac{1}{x+1} + \ C$$

Volviendo a la integral original

$$I = x - \left(-\frac{1}{x+1}\right) + C \quad = x + \frac{1}{x+1} + C$$

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$$\int_0^{u>2} sen^3 \cos x \, dx \quad = 1$$

Sea: $u = sen\, x$

$du = \cos x \, dx$

Los nuevos límites de integración

- ✓ Límite inferior

$Si\ x = 0 \qquad u = sen\,(0) = 0$

- ✓ Límite superior

$Si\ x = \frac{\pi}{2} \qquad u = sen\,\left(\frac{\pi}{2}\right) = 1$

Por lo tanto

$$I = \int_0^1 u^3 du \quad = \quad \frac{u^4}{4}\int_0^1 \left(\frac{1}{4} - 0\right)$$

$$I = \frac{1}{4}$$

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$$\int \frac{2+\sqrt{x}}{1-\sqrt{x}}\, dx \quad = \quad I$$

Sea: $u = 1 - \sqrt{x}$

$\sqrt{x} = 1 - u \quad = \qquad x = (1-u)^2$

$dx = 2(1-u)(-1)du \ = \ 2(1-u)du$

Ahora, $2 + \sqrt{x} = 2 + (1-u) \ = \ 3 - u$

Al integrar

$$I = \int \left(\frac{3-u}{u}\right)[-2(1-u)]\, du$$

$$I = -\int \left(\frac{(3-u)(1-u)}{u}\right) du$$

$$I = -\int \left(\frac{3-3u-u+u^2}{u}\right) du$$

$$= -\int \left(\frac{3-4u+u^2}{u}\right) du$$

$$= -(3\int \frac{du}{u} - 4\int du + \int u\, du)$$

$$= -\left(3\lim(u) - 4u + \frac{u^2}{2}\right) + C$$

$$= -\left(3\lim(u) - 4u + \frac{u^2}{2}\right) + C$$

$$I = \mathcal{X} + \frac{1}{2} dm|x^2 + 1| + C$$

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$$\int \frac{2\chi}{(x + 1)^2}\, dx \quad = I$$

sea